Damage spreading in the Ising model

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We present two interesting results regarding damage spreading in ferromagnetic Ising models. First, we show that a damage spreading transition can occur in an Ising chain that evolves in contact with a thermal reservoir. Damage heals at low temperature and spreads at high T. The dynamic rules for the system's evolution for which such a transition is observed are as legitimate as the conventional rules (Glauber, Metropolis, heat bath). Our second result is that such transitions are not always in the directed percolation universality class. [S1063-651X(97)13606-6]

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I. INTRODUCTION

A system is said to exhibit damage spreading (DS) if the ''distance'' between two of its replicas, that evolve under the same thermal noise but from slightly different initial conditions, increases with time. Even though DS was first introduced in the context of biologically motivated dynamical systems [1], it has evolved into an important tool in physics. It is used in equilibrium [2] for measuring accurately dynamic exponents and also out of equilibrium, to study the influence of initial conditions on the temporal evolution of various systems. In particular, one hoped that DS could be used to identify ''phases'' of *chaotic* behavior in systems with no intrinsic dynamics, such as Ising ferromagnets [3,4] and spin glasses [5]. Such hopes were dampened when it was realized that different algorithmic implementations of the same physical system's dynamics (such as Glauber versus heat bath or Metropolis Monte Carlo) can have different DS properties [6,7]. This implies that DS is not an intrinsic property of a system [8], since two equally legitimate algorithms yield contradictory results. This problem was addressed recently in Ref. [9], where we realized that one *can* define ''phases'' on the basis of their DS properties in an algorithm-independent manner. To do this one must, however, consider simultaneously the *entire set* $\mathcal{A}$ of possible algorithms (dynamic procedures) that are consistent with the physics of the model studied (such as detailed balance, interaction range, and symmetries). Every system must belong to one of three possible DS phases, depending on whether damage spreads for all, none or a part of the members of the set $\mathcal{A}$.

Once we have been led to consider a large family of algorithms, it was natural to revisit an old question, such as the possibility for DS in the one-dimensional (1D) Ising ferromagnet. In this case all conventional dynamic procedures agree that damage does not spread. We show here that once the family of dynamic procedures is extended in the spirit explained above, *a DS transition is possible in the 1D Ising model.* Having found such a DS transition, it is again natural to investigate to which universality class it belongs. So far this issue could be addressed only for the 2D case; since it is much easier to obtain high-quality numerical data in one dimension, we were able to test carefully a conjecture of Grassberger [8], according to which the generic universality class of damage spreading transitions is directed percolation (DP). This indeed is correct, but we discovered that if the dynamics that is being used has certain symmetries, *the DS transition is not in the DP class*. Interestingly this is the case for Glauber dynamics of the $H=0$ Ising model, for which the DS transition is non-DP.

We start by reviewing briefly [6,7] the conventional algorithms—Glauber, heat bath (HB), and Metropolis—and show that they form a particular subset of some general set of legitimate rules $\mathcal{A}$. All members of $\mathcal{A}$ satisfy detailed balance with respect to the same Hamiltonian; hence all these rules generate the same equilibrium ensemble as the conventional algorithms and are equally legitimate to mimic the temporal evolution of an Ising system in contact with a thermal reservoir. Next, we introduce two ''new'' dynamic rules, which constitute just another subset of $\mathcal{A}$, and show that for these two rules a DS transition *does* occur in the 1D Ising model. Moreover, as we show in the example of the second rule, an additional Z_2 symmetry of the DS order parameter leads to a transition that is not in the DP universality class.

II. PREVIOUS WORK, WITH CONVENTIONAL ALGORITHMS

Denote the site which is being updated by i, and the set of its neighbors by j. The energy at time t is given by

$$\frac{\mathcal{H}}{k_BT} = -\sum_i h_i(t)\sigma_i(t), \quad h_i(t) = \sum_j K_{ij}\sigma_j(t), \tag{1}$$

where $K_{ij}=J/k_BT$ and $\sigma_i(t)=\pm 1$. Define a transition probability $p_i(t)$

$$p_i(t) = \frac{e^{h_i(t)}}{e^{h_i(t)}+e^{-h_i(t)}}. \tag{2}$$

The update rules of HB, Glauber, and Metropolis dynamics are expressed in terms of random numbers $z=z_i(t)$, selected with equal probability from the interval [0,1]. The rule for a *standard HB* is

$$\sigma_i(t+1) = \text{sgn}[p_i(t)-z]. \tag{3}$$

A different dynamic process is obtained by generating at each site *two independent* random numbers, z_+ and z_-, and using the first if $\sigma_i(t)=+1$ and the second when $\sigma_i(t)=-1$. The rules of this *uncorrelated HB* dynamics may be written as

$$\sigma_i(t+1)=\begin{cases}\mathrm{sgn}[p_i(t)-z_+] & \text{if } \sigma_i(t)=+1\\ \mathrm{sgn}[p_i(t)-z_-] & \text{if } \sigma_i(t)=-1.\end{cases} \tag{4}$$

Glauber dynamics uses only one random number per site

$$\sigma_i(t+1)=\begin{cases}+\mathrm{sgn}[p_i(t)-z] & \text{if } \sigma_i(t)=+1\\ -\mathrm{sgn}[1-p_i(t)-z] & \text{if } \sigma_i(t)=-1.\end{cases} \tag{5}$$

This rule can be expressed in the form of Eq. (4), but with the two random numbers completely *anticorrelated*, i.e., $z_+ + z_- = 1$.

Finally the rules for *Metropolis* dynamics read

$$\sigma_i(t+1)=\begin{cases}+\mathrm{sgn}[p_i^+(t)-z] & \text{if } \sigma_i(t)=+1\\ -\mathrm{sgn}[p_i^-(t)-z] & \text{if } \sigma_i(t)=-1,\end{cases} \tag{6}$$

where $p_i^{\pm}(t)=\min(1,e^{\mp 2h_i(t)})$.

It is easy to show that given $\sigma_{i-1}(t),\sigma_i(t),\sigma_{i+1}(t)$, the probability to obtain $\sigma_i(t+1)=+1$ is the same for standard HB, uncorrelated HB, and Glauber dynamics [10]. Hence, by observing the temporal evolution of a *single* Ising system, one cannot tell by which of these methods was its trajectory in configuration space generated. The difference between these dynamics may become evident only when we observe the evolution of two replicas, i.e., study damage spreading. Indeed Stanley *et al.* [4] and also Mariz, Herrmann, and de Arcangelis [6] found, using Glauber dynamics, that damage spreads for the 2D Ising model for $T>T_c$; similarly for Metropolis dynamics [6]. More recently Grassberger [11] claimed that the DS transition occurs slightly below T_c for Glauber dynamics, which was also observed in the corresponding mean-field theory [12]. On the other hand, damage does not spread at any temperature with standard HB dynamics for neither the 2D [6] nor 3D Ising models [5]. The 3D model did exhibit DS for $T>T^*$ with $T^*<T_c$ when Metropolis [13] and Glauber [11,14] dynamics were used. In the 1D Ising model with HB, Glauber, or Metropolis dynamics, no damage spreading was observed.

III. GENERAL CLASS OF DYNAMIC PROCEDURES FOR THE ISING MODEL

The dynamic rules considered here for the 1D Ising model consist of local updates, where a random variable $r=\pm 1$ is assigned to the spin σ_i:

$$\sigma_i(t+1):=r_{\sigma_{i-1}(t),\sigma_i(t),\sigma_{i+1}(t)}\,. \tag{7}$$

This random variable is generated in some probabilistic procedure using one or several random numbers. As in the conventional algorithms discussed above, we allow the random variable to depend only on the values taken at time t by the updated spin itself and the spins with which it interacts (i.e., its nearest neighbors). The set of all one-point functions $\langle r_{\sigma_{i-1},\sigma_i,\sigma_{i+1}}\rangle$ determines the transfer matrix of a *single* system. Here $\langle\ \rangle$ denotes the average over many independent realizations of random numbers. The simultaneous evolution (and, hence, DS) of *two replicas* $\{\sigma\}$ and $\{\sigma'\}$ is, however, governed by a joint transfer matrix of the two systems which, in turn, is completely determined by the two-point functions $\langle r_{\sigma_{i-1},\sigma_i,\sigma_{i+1}} r_{\sigma'_{i-1},\sigma'_i,\sigma'_{i+1}}\rangle$. In general, n-point functions determine the joint transfer matrix of n replicas. An important requirement is that all correlation functions have to be invariant under the symmetries of the model [9]. For a homogeneous Ising chain in zero field these symmetries are invariance under reflection,

$$\langle r_{\sigma_{i-1},\sigma_i,\sigma_{i+1}}\rangle=\langle r_{\sigma_{i+1},\sigma_i,\sigma_{i-1}}\rangle, \tag{8}$$

$$\langle r_{\sigma_{i-1},\sigma_i,\sigma_{i+1}} r'_{\sigma'_{i-1},\sigma'_i,\sigma'_{i+1}}\rangle=\langle r_{\sigma_{i+1},\sigma_i,\sigma_{i-1}} r'_{\sigma'_{i+1},\sigma'_i,\sigma'_{i-1}}\rangle,$$

and global inversion of all spins (Z_2 symmetry),

$$\langle r_{\sigma_{i-1},\sigma_i,\sigma_{i+1}}\rangle=-\langle r_{-\sigma_{i-1},-\sigma_i,-\sigma_{i+1}}\rangle, \tag{9}$$

$$\langle r_{\sigma_{i-1},\sigma_i,\sigma_{i+1}} r'_{\sigma'_{i-1},\sigma'_i,\sigma'_{i+1}}\rangle =\langle r_{-\sigma_{i-1},-\sigma_i,-\sigma_{i+1}} r'_{-\sigma'_{i-1},-\sigma'_i,-\sigma'_{i+1}}\rangle.$$

For both HB and for Glauber dynamics, the one-point functions are given by

$$\langle r_{\sigma_{i-1},\sigma_i,\sigma_{i+1}}\rangle=2p_i-1. \tag{10}$$

The corresponding transfer matrices for single systems are, hence, identical. On the other hand, the two-point functions for HB and Glauber dynamics are different, so that damage evolves differently (see Table I). Still, damage does not spread in one dimension for any of these algorithms at any temperature.

IV. DYNAMIC RULE FOR WHICH DAMAGE DOES SPREAD IN 1D

Consider the following dynamics for the 1D Ising model

$$r_{\sigma_{i-1},\sigma_i,\sigma_{i+1}}=\begin{cases}+\mathrm{sgn}(p_i-z) & \text{if } \sigma_{i-1}=\sigma_{i+1}\\ -\mathrm{sgn}(1-p_i-z) & \text{if } \sigma_{i-1}\neq\sigma_{i+1}.\end{cases} \tag{11}$$

As can be checked easily, this dynamical rule yields the same one-point correlations as in Eq. (10). Therefore, the evolution of a single replica using this rule cannot be distinguished from that of Glauber or HB dynamics. However, the two-point correlations (and therewith damage spreading properties) are different (see Table I). Unlike Glauber and HB, this dynamics does exhibit a damage spreading transition in one dimension. This can be seen as follows. At $T=\infty$, Eq. (11) reduces to

$$r_{\sigma_{i-1},\sigma_i,\sigma_{i+1}}=\sigma_{i-1}\sigma_{i+1}\mathrm{sgn}(\tfrac{1}{2}-z), \tag{12}$$

which implies that the local damage $\Delta_i(t)=1-\delta_{\sigma_i(t),\sigma'_i(t)}$ evolves deterministically:

$$\Delta_i(t+1)=\begin{cases}0 & \text{if } \Delta_{i-1}(t)=\Delta_{i+1}(t)\\ 1 & \text{if } \Delta_{i-1}(t)\neq\Delta_{i+1}(t).\end{cases} \tag{13}$$

TABLE I. Two-point correlations in the one-dimensional Ising model for various dynamic rules. We used the notation $\kappa = \tanh 2J/k_BT$.

Correlation function	Glauber dyn.	Usual HB	Uncorr. HB	Dynamics of Eq. (11)	Dynamics of Eq. (18)
$\langle r_{---}\, r_{--+}\rangle$	$1-\kappa$	$1-\kappa$	$1-\kappa$	$\kappa-1$	$\lambda(1-\kappa)$
$\langle r_{---}\, r_{-+-}\rangle$	$2\kappa-1$	1	κ^2	1	$2\kappa-1$
$\langle r_{---}\, r_{-++}\rangle$	$\kappa-1$	$1-\kappa$	0	$\kappa-1$	$\lambda(\kappa-1)$
$\langle r_{---}\, r_{+-+}\rangle$	$1-2\kappa$	$1-2\kappa$	$1-2\kappa$	$1-2\kappa$	$1-2\kappa$
$\langle r_{---}\, r_{+++}\rangle$	-1	$1-2\kappa$	$-\kappa^2$	$1-2\kappa$	-1
$\langle r_{--+}\, r_{-+-}\rangle$	$\kappa-1$	$1-\kappa$	0	$\kappa-1$	$\lambda(\kappa-1)$
$\langle r_{--+}\, r_{-++}\rangle$	-1	1	0	1	-1
$\langle r_{--+}\, r_{+--}\rangle$	1	1	1	1	1
$\langle r_{--+}\, r_{+-+}\rangle$	$1-\kappa$	$1-\kappa$	$1-\kappa$	$\kappa-1$	$\lambda(1-\kappa)$
$\langle r_{--+}\, r_{++-}\rangle$	-1	1	0	1	-1
$\langle r_{-+-}\, r_{+-+}\rangle$	-1	$1-2\kappa$	$-\kappa^2$	$1-2\kappa$	-1

Since this is exactly the update rule of a Domany-Kinzel model [15] in the active phase (with $p_1=1$ and $p_2=0$), we conclude that for $T=\infty$ damage spreads. On the other hand, for $T=0$, Eq. (11) reduces to

$$r_{\sigma_{i-1},\sigma_i,\sigma_{i+1}} = \begin{cases} \sigma_{i-1} & \text{if } \sigma_{i-1}=\sigma_{i+1} \\ \mathrm{sgn}(z-\tfrac{1}{2}) & \text{if } \sigma_{i-1}\neq\sigma_{i+1}. \end{cases} \tag{14}$$

In this case damage evolves probabilistically, and cannot be viewed as an independent process. One can, however, show that the expectation value to obtain damage at site i, averaged over many realizations of random numbers, satisfies the inequality $\langle\Delta_i(t+1)\rangle \leq \frac{1}{2}\langle\Delta_{i-1}(t)+\Delta_{i+1}(t)\rangle$, that is, $\langle\Delta(t+1)\rangle \leq \langle\Delta(t)\rangle$. This means that for $T=0$ damage does not spread. In fact, simulating the spreading process, one observes a DS transition at finite temperature. A typical temporal evolution near the transition is shown in Fig. 1.

In order to determine the critical exponents that characterize the DS transitions, we perform dynamic Monte Carlo simulations [16]. Two replicas are started from identical random initial conditions, where one damaged site is inserted at the center. Both replicas then evolve according to the dynamic rules of the system using the same set of random numbers. In order to minimize finite-size effects, we simulate a large system of 5000 sites with periodic boundary conditions. For various temperatures we perform 10^6 independent runs up to 1500 time steps. However, in many runs damage heals very soon, so that the run can be stopped earlier. As usual in this type of simulations, we measure the survival probability $P(t)$, the number of damaged sites $\Delta(t)$, and the mean-square spreading of damage from the center $R^2(t)$ averaged over the active runs. At the DS transition, these quantities are expected to scale algebraically in the large time limit

$$P(t)\sim t^{-\delta}, \quad \Delta(t)\sim t^{\eta}, \quad R^2(t)\sim t^z. \tag{15}$$

The critical exponents δ, η, and z are related to the density exponent β and the scaling exponents $\nu_\perp$ and $\nu_{||}$ by $\delta=\beta/\nu_{||}$ and $z=2\nu_\perp/\nu_{||}$, and obey the hyperscaling relation $4\delta+2\eta=dz$. At criticality, the quantities (15) show straight lines in double logarithmic plots. Off criticality, these lines are curved. Using this criterion we estimate the critical temperature for the DS transition by $J/k_BT^*=0.2305(5)$. The exponents δ, η, and z are measured at criticality, while the density exponent β is determined off criticality by measuring the stationary Hamming distance $\Delta(T)\sim(T-T^*)^\beta$ in the spreading phase. The results of our simulations are shown in Fig. 2. From the slopes in the double logarithmic plots we obtain the estimates $\delta=0.165(5)$, $\eta=0.315(10)$, $z=1.29(3)$, and $\beta=0.26(2)$ which are in fair agreement with the known [17] exponents for directed percolation $\delta=0.15947(3)$, $\eta=0.31368(4)$, $z=1.26523(4)$, and $\beta=0.27649(4)$. We therefore conclude that, in agreement with Grassberger's conjecture [8], the DS transition belongs to the DP universality class. This is very plausible; as far as the damage variable is concerned there is a *single absorbing state* (of no damage at all) and the transition is from a phase in which the system ends up in this state to one in which it does not, just as is the case for DP.

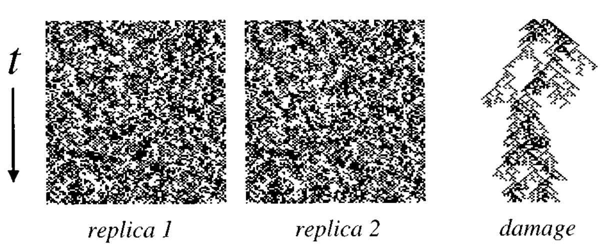

FIG. 1. Temporal evolution of damage in the one-dimensional Ising model of size 200 with the dynamics of Eq. (11) near the DS transition $J/k_BT^*=0.2305$. Each configuration is represented by a row of pixels, and time goes downwards. The two replicas start from identical initial conditions. At an early time, a damage of five sites is inserted in the center.

V. DAMAGE SPREADING TRANSITION WITH NON-DP EXPONENTS

Different critical properties are expected [18–23] for rules with two distinct absorbing states (of the damage variables) related by symmetry. It is important to note that the Z_2 symmetry of the Ising system does not suffice—inverting all spins in *both* replicas does not change the damage variable

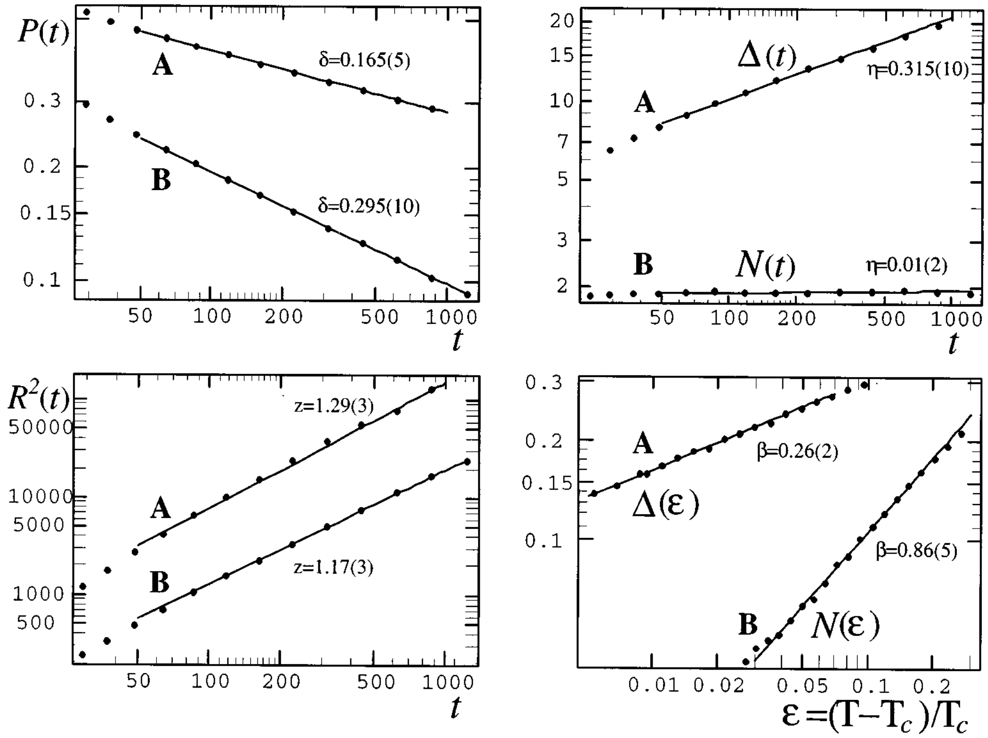

FIG. 2. Numerical results for the one-dimensional Ising model with (A) the dynamics of Eq. (11) and (B) the dynamics of Eq. (18). The measured quantities are explained in the text.

(the Hamming distance between the two configurations). Therefore, we are looking for dynamic rules which (a) have two types of absorbing states—one with no damage and the other with full damage. Furthermore (b), the two play completely symmetric roles. One can see that both (a) and (b) hold for rules that satisfy the condition

$$r_{\sigma_{i-1},\sigma_i,\sigma_{i+1}} = -r_{-\sigma_{i-1},-\sigma_i,-\sigma_{i+1}}. \tag{16}$$

The immediate consequence of this condition is that if a configuration $\{\sigma(t)\}$ evolves in one time step into $\{\sigma(t+1)\}$, then the spin-reversed configuration $\{-\sigma(t)\}$ will evolve into precisely $\{-\sigma(t+1)\}$. Imagine now simultaneous evolution of two replicas with initial states $\{\sigma\}$ and $\{\sigma'\}$, giving rise to a damage field $\{\Delta\}$. Reversal of the initial state on *one* of the replicas will give sign-reversed spin states on this replica, and hence the damage field $\{-\Delta\}$ will evolve. Thus, for rules that satisfy condition (16), the *damage variable* has a Z_2 symmetry. A particular consequence of this symmetry is that if two initial states are the exact sign reverse of one another, this will persist at all subsequent times. Therefore, inasmuch as $\Delta=0$ (no damage) is an absorbing state, so is the situation of full damage, $\Delta=1$. For systems with such Z_2 symmetry we expect the DS transition (if it exists) to exhibit non-DP behavior.

It is quite remarkable to note that Glauber dynamics satisfies Eq. (16). The Z_2 symmetry of damage in the 1D Glauber model is illustrated in Fig. 3(a). One can see that compact islands of damaged sites are formed because damage does not heal spontaneously inside such islands but only at the edges. However, as mentioned earlier, there is no DS transition in the 1D Glauber model.

Consider now a different dynamic rule

$$r_{\sigma_{i-1},\sigma_i,\sigma_{i+1}} = \begin{cases} +\mathrm{sgn}(p_i - z) & \text{if } \sigma_{i-1}\sigma_i\sigma_{i+1}=1 \\ -\mathrm{sgn}(1-p_i-z) & \text{if } \sigma_{i-1}\sigma_i\sigma_{i+1}=-1. \end{cases} \tag{17}$$

For this rule, which also satisfies Eq. (16), we observe in simulations that damage always spreads [see Fig. 3(c)]. In order to generate a Z_2-symmetric DS transition in one dimension, we use a rule that interpolates between this and Glauber. This can be done by introducing a second parameter $0 \leq \lambda \leq 1$, and ''switching'' between Glauber dynamics and rule (17) as follows: in each update an additional random

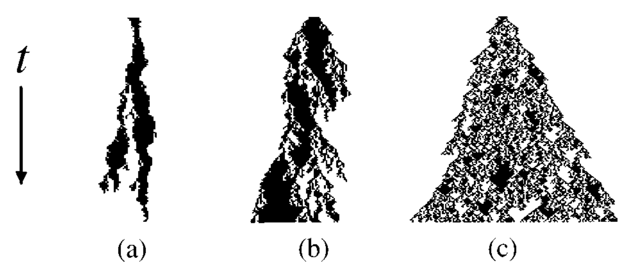

FIG. 3. Z_2-symmetric damage spreading transition. Two replicas with 200 sites are started from identical random initial conditions. At an early time five damaged sites are introduced in the center. For fixed temperature $J/k_BT=0.25$, a typical temporal evolution of damage is shown for (a) Glauber dynamics $\lambda=1$, (b) near the transition $\lambda^*=0.82$, and (c) in the spreading regime $\lambda=0$. Because of the symmetry, islands of damaged sites can heal only at the edges.

number $\tilde{z}$ is generated. If $\tilde{z} \geq \lambda$, rule (17) is applied, otherwise Glauber dynamics is used. This mixed dynamics can be expressed as

$$r_{\sigma_{i-1},\sigma_i,\sigma_{i+1}} = \begin{cases} +\mathrm{sgn}(p_i - z) & \text{if } y = 1 \\ -\mathrm{sgn}(1 - p_i - z) & \text{if } y = -1, \end{cases} \quad (18)$$

where

$$y = \tfrac{1}{2}\sigma_i[(1+\sigma_{i-1}\sigma_{i+1}) + (1-\sigma_{i-1}\sigma_{i+1})\mathrm{sgn}(\lambda - \tilde{z})].$$

Again this rule leads to the one-point correlations of Eq. (10), i.e., the temporal evolution of a single replica is the same as in Glauber and HB dynamics. However, varying λ (at fixed T) we find a critical value λ^* where a DS transition occurs. A typical temporal evolution of damage near the transition is shown in Fig. 3(b). Since ''damage'' and ''no damage'' play a symmetric role, the Hamming distance Δ (the density of damaged sites) cannot be used as an order parameter. Instead one has to use the density of *kinks* N (domain walls) between damaged and healed domains. By definition, the number of kinks is conserved modulo 2 which establishes a parity conservation law. As can be seen in Fig. 3, two processes compete with each other: kinks annihilate mutually $(2X \to 0)$, and already existing kinks branch into an odd number of kinks $(X \to 3X, 5X, \dots)$. Both processes resemble a branching annihilating walk with an even number of offspring. This branching process has a continuous phase transition that belongs to the so-called parity-conserving (PC) universality class. Phase transitions of this type have been observed in a variety of models, including certain probabilistic cellular automata [18], nonequilibrium kinetic Ising models with combined zero- and infinite-temperature dynamics [19], interacting monomer-dimer models [20], branching-annihilating random walks [21], and certain lattice models with two absorbing states [22]. In all these models the symmetry appears either as a parity conservation law or as an explicit Z_2 symmetry among different absorbing phases. A field theory describing PC transitions is currently developed in Ref. [23].

The PC universality class is characterized by exponents $\delta = 0.285(5)$, $\eta = 0.00(1)$, $z = 1.15(1)$, and $\beta = 0.92(2)$. In fact, repeating the numerical simulations described above for $J/k_BT = 0.25$ and $\lambda^* = 0.82(1)$ (see Fig. 2), we obtain the estimates $\delta = 0.295(10)$, $\eta = 0.01(2)$, $z = 1.17(3)$, and $\beta = 0.86(5)$, which are in fair agreement with the known values. We therefore conclude that the DS transition observed for the dynamics of Eq. (18) belongs to the PC universality class. Furthermore, our findings imply that the DS transitions observed [11] for the 2D Ising model with Glauber dynamics should also exhibit PC exponents (remember: $d = 2$) in zero field, and *cross over* to (2D) DP values when a field is switched on.

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